On the Equivalence of Automaton-based Representations of Time Granularities

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Motivations

- **relational databases**:
  to express temporal information at different time granularities, to relate different granules and convert associated data (queries)
- **artificial intelligence**:
  to reason about temporal relationships, e.g, to check consistency, validity, and equivalence of temporal constraints at different time granularities (temporal CSPs)
- **data mining**:
  to discover temporal relationships between collected events, to derive implicit information from such relationships.

Outline

- Introduction (time granularities and their representations)
- The automaton-based approach (Single-string automata)
- The equivalence problem
- The solution for RLA-based representations

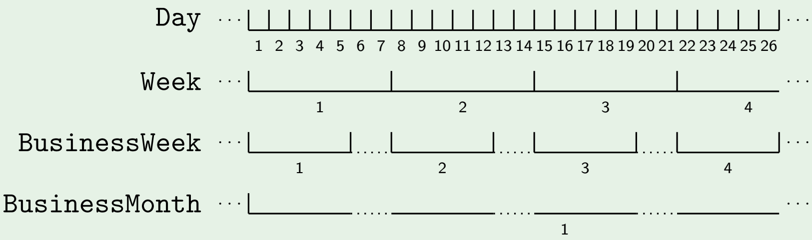
Let $(\mathbb{N}^+, <)$ be the underlying **temporal domain**.

Definition

A **time granularity** G is a partition of a *subset* of $(\mathbb{N}^+, <)$ such that, for every pair of distinct sets $g, g' \in G$ (called **granules**), one of the following two conditions holds:

1. $g < g'$ (i.e., for all $t \in g$ and for all $t' \in g'$, $t < t'$),
2. $g > g'$ (i.e., for all $t \in g$ and for all $t' \in g'$, $t > t'$).

Examples



We cannot finitely represent *all granularities over an infinite domain*
$\Rightarrow$ we have to restrict ourselves to a proper subclass of structures.

Possible approaches to time granularity representation

- **algebraic one**:
  relationships between granularities are represented
  by algebraic terms built up from a finite set of operators
  (e.g., $\texttt{Week} = \mathit{Group}_7(\texttt{Day})$ in the Calendar Algebra)

  C. Bettini, S. Jajodia, S.X. Wang. *Time Granularities in Databases, Data Mining, and Temporal Reasoning.* 2000.

- **logical one**:
  time granularities are defined by models of
  formulas in a suitable language (e.g., PLTL)

  C. Combi, M. Franceschet, A. Peron. *Representing and Reasoning about Temporal Granularities.* Journal of Logic and Computation, 2004.

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Possible approaches to time granularity representation

- **string-based one**:
  relationships between time points and granules are encoded by sequences of symbols from a given alphabet (e.g., Granspecs)

  J. Wijsen. *A String-based Model for Infinite Granularities.* Proceedings of the AAAI Workshop on Spatial and Temporal Granularities, 2000.

- **automaton-based one**:
  automata are used to encode string-based representations of time granularities (e.g., Single-string Automata)

  U. Dal Lago, A. Montanari. *Calendars, Time Granularities, and Automata.* Proceedings of the 7th International Symposium on Spatial and Temporal Databases, 2001.

Basic ingredients of the string-based approach

- A fixed alphabet $\{\blacksquare, \square, \blacktriangleleft\}$, where
  $\blacksquare$ represents time points covered by some granule,
  $\square$ represents gaps within and between granules,
  $\blacktriangleleft$ represents the last time point of each granule
- Restriction to **ultimately periodic words** over $\{\blacksquare, \square, \blacktriangleleft\}$, namely, to finite granularities or granularities that, ultimately, periodically group instants of the temporal domain.

Example

The infinite word $\blacksquare\blacksquare\blacksquare\blacksquare\blacktriangleleft\square\square\blacksquare\blacksquare\blacksquare\blacksquare\blacktriangleleft\square\square\blacksquare\blacksquare\blacksquare\blacksquare\blacktriangleleft\square\square\ldots$ represents the granularity `BusinessWeek` in terms of `Day`.

Such a string can be finitely presented by a **Granspec**, namely, a pair *prefix-pattern*, such as $(\varepsilon, \blacksquare\blacksquare\blacksquare\blacksquare\blacktriangleleft\square\square)$.

Connection between ultimately periodic words and automata:

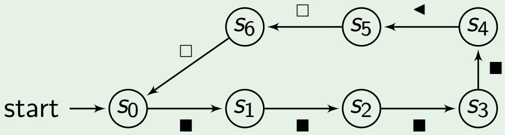
Proposition

Any ultimately periodic word is recognized by a **Single-string Automaton** (**SSA**), namely, a Büchi automaton accepting a single infinite word.

Corollary

Finite granularities and *ultimately periodical granularities* can be represented by Single-string Automata.

An SSA representing `BusinessWeek`



Problem

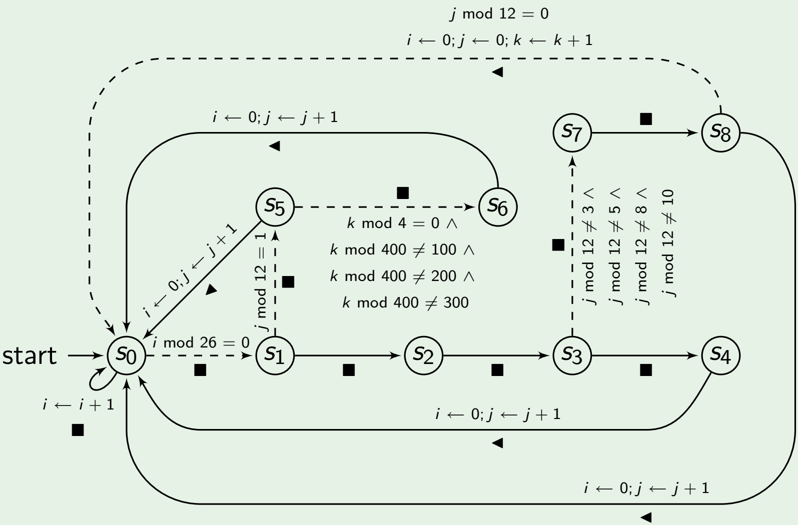
Representations based on Granspecs and SSA are *too large* with respect to inherently simple structure of granularities.

Possible solution

Use counters and multiple transitions to compactly encode redundancies of time granularities:

- **counters** range over *discrete domains* (e.g., $\mathbb{N}$),
- **update operators** modify the values of the counters,
- **guards** rule the activation of **primary transitions** and **secondary transitions** (note: *only one transition is enabled at each step*).

An Extended SSA representing Month



New problem

Extended SSA *do not ease algorithmic manipulation*.

Solution (**Restricted Labeled Single-string Automata - RLA**)

One can introduce suitable restrictions:

- states can be **labeled** (namely, they recognize a symbol)

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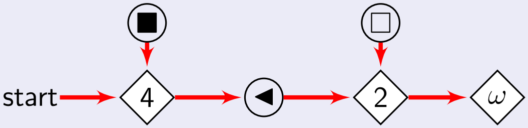

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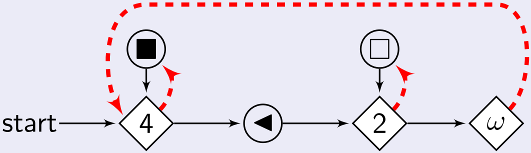
- secondary transitions depart from non-labeled states and form **back edges** in the forest graph,



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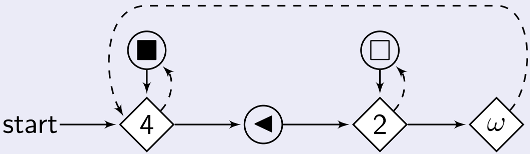
- the graph of primary transitions is *acyclic* (**forest graph**),
- secondary transitions depart from non-labeled states and form **back edges** in the forest graph,
- *uniform policy* of counter update (decrement/reset).



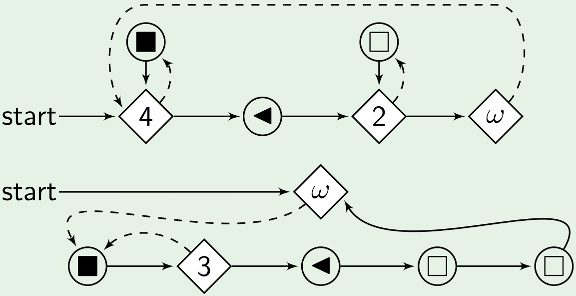
Definition

The **equivalence problem** consists in deciding whether two given representations define *the same time granularity*.

Examples

The two Granspecs (ε, ■■■■◀□□) and (■■, ■■◀□□■■■■◀□□) are equivalent.

Similarly, the two Restricted Labeled SSA are equivalent:



As for string-based specification

The equivalence problem reduces to the *pattern matching problem*

⇒ the algorthm is linear in the size of the input Granspecs.

As for automaton-based representations

Trivial (but *inefficient*) solutions exist:

simply unfold the automata into equivalent Granspecs and then use pattern matching algorithms to test equivalence

⇒ exponential complexity for both Extended Single-String Automata and Restricted Labeled Single-String Automata.

As for Extended SSA

A better (but still rather inefficient) solution exists:

the equivalence problem is reduced to the *satisfiability problem for PLTL*** (i.e., a temporalization of a fragment of Presburger logic)

⇒ the problem turns out to be in PSPACE
(*completeness* proved by using a reduction from the satisfiability problem for quantified boolean formulas).

S. Demri. *LTL Over Integer Periodicity Constraints.* Proceedings of the 7th International Conference on Foundations of Software Science and Computation Structures, 2004.

In the following, we focus on a solution to the equivalence problem for Restricted Labeled Single-String Automata ...

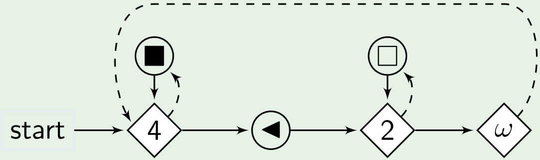
Definition

A **chain** is a path of primary transitions that goes

1. either *from the target to the source of a secondary transition*

Example

Consider the following automaton:



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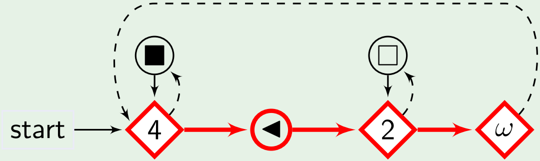

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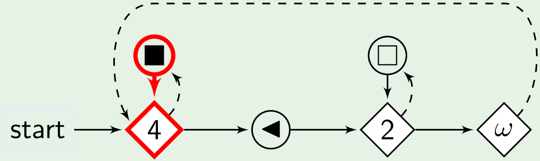

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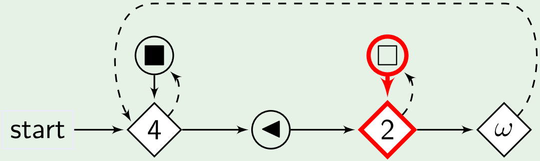

Definition

A **chain** is a path of primary transitions that goes

1. either *from the target to the source of a secondary transition*
2. or *from the entry point "start" to the deepest state*.

Example

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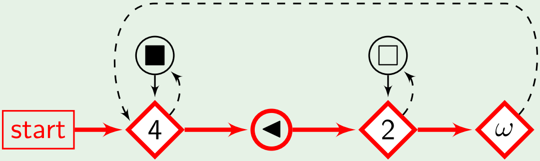

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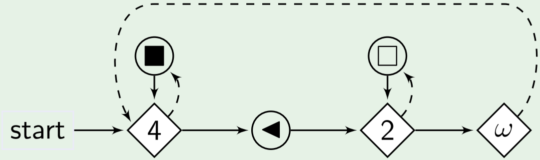
A **chain** is a path of primary transitions that goes

1. either *from the target to the source of a secondary transition*
2. or *from the entry point "start" to the deepest state*.

An automaton is **sharing** if it contains some *overlapping chains*.

Example

Consider the following automaton:



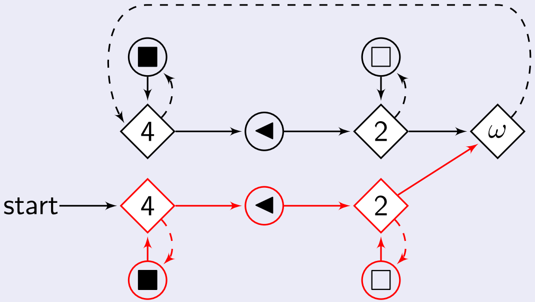
...it is easily seen to be sharing.

Lemma

Any Restricted Labeled SSA can be transformed into an equivalent non-sharing automaton with (at most) a polynomial blowup of states.

Proof idea

Simply duplicate overlapping portions of chains:



Fact

Two Restricted Labeled SSA $\mathcal{A}$ and $\mathcal{B}$ are *not equivalent* iff there exist two distinct symbols a, b such that

$$Occ_{\mathcal{A}}(a) \cap Occ_{\mathcal{B}}(b) \neq \emptyset$$

where $Occ_{\mathcal{A}}(a)$ denotes the (possibly infinite) set of *occurrence positions of a* in the word recognized by $\mathcal{A}$.

Proposition

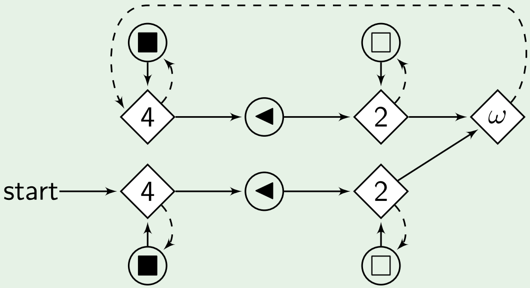
If $\mathcal{A}$ is *non-sharing*, then the set $Occ_{\mathcal{A}}(a)$ can be presented as a **finite union of linear progressions** of the form

$$p_1 C_1 + \ldots + p_n C_n$$

where $p_i \in \mathbb{N}^+$ and C_i is an interval of $\mathbb{N}$ (the presentation uses only *polynomial size* w.r.t. the size of the automaton).

Example

Consider the non-sharing Restricted Labeled SSA $\mathcal{A}$:



$$Occ_{\mathcal{A}}(\blacksquare) = \underbrace{1}_{\text{first position}} + \underbrace{1}_{\text{loop length}} \cdot \underbrace{[0,3]}_{\text{counter interval}}$$

$$\cup \; \underbrace{8}_{\text{first position}} + \underbrace{1}_{1^{st}\text{ loop length}} \cdot \underbrace{[0,3]}_{1^{st}\text{ counter interval}} + \underbrace{7}_{2^{nd}\text{ loop length}} \cdot \underbrace{[0,\omega[}_{2^{nd}\text{ counter interval}}$$

Fact

Testing the intersection of two linear progressions

$$p_1 C_1 + \ldots + p_n C_n \quad \text{and} \quad q_1 D_1 + \ldots + q_m D_m$$

is equivalent to the problem of testing the satisfiability of the **linear diophantine equation**

$$p_1 x_1 + \ldots + p_n x_n \; - \; q_1 y_1 - \ldots - q_m y_m \; = \; 0$$

over the **bounded variables** $min(C_i) \leq x_i \leq max(C_i)$, $min(D_j) \leq y_j \leq max(D_j)$.

Theorem

The non-equivalence problem for Restricted Labeled SSA is reducible to the satisfiability problem for linear diophantine equations with bounds on variables.

$\Rightarrow$ *The RLA equivalence problem is in Co-NP.*

Open problem

Establish whether the non-equivalence problem for Restricted Labeled Single-string Automata is *Co-NP-complete* or not.

(Note: it is conceivable that the problem may enjoy a *deterministic polynomial-time* solution)

As a matter of fact, Restricted Labeled Single-string automata turned out to be well suited to algorithmic manipulation:

- polynomial-time algorithms for *searching symbol occurrences* in the word recognized by an RLA,
- polynomial-time algorithms that compute *granule conversions* between different time granularities w.r.t. to meaningful relationships (e.g., `intersect`, `cover`, `covered by`),
- polynomial-time algorithms that compute the *most compact* representation (or the *most tractable* representation) of a given string-based specification.